Exercise 1  Consider the nonlinear dynamical system (in polar coordinates)
\[
\begin{align*}
\dot{r} &= r - r^2 \\
\dot{\theta} &= 1
\end{align*}
\]
a) Determine the Poincaré map from $S$ into itself, where $S$ is the positive real axis.
b) Show that the system has a unique periodic orbit and classify its stability.
c) Find the characteristic multiplier of the periodic orbit, i.e., the Floquet multiplier.

Exercise 2  Consider the nonlinear dynamical system
\[
\begin{align*}
\dot{x} &= xy - x^2 - x \\
\dot{y} &= x^2 - y + \mu
\end{align*}
\]
a) Sketch the nullclines and determine the fixed points as a function of $\mu$.
b) Find and classify the bifurcations that occur as $\mu$ varies.
c) (Non-mandatory computer work) Plot several sections of the phase portrait before and after the bifurcation points. (you are welcome to use the Matlab code available in the course website).

Exercise 3  Consider the predator-prey model
\[
\begin{align*}
\dot{x} &= x^2(1 - x) - xy \\
\dot{y} &= xy - \mu y \quad \mu \geq 0
\end{align*}
\]
where $x, y \geq 0$ are the dimensionless populations of preys and predators, respectively.
a) Determine and classify all fixed points as a function of $\mu$.
b) Show that a Hopf bifurcation occurs at $\mu = 1/2$. At which point in the phase plane?
c) (Non-mandatory computer work) Plot several sections of the phase portrait for $\mu$ in the range $0 < \mu < 2$. (you are welcome to use the Matlab code available in the course website). Is the Hopf bifurcation at $\mu = 1/2$ subcritical or supercritical? What happens to the population of predators for $\mu > 1$?